TOWNSHIP OF UNION PUBLIC SCHOOLS

UHS – Algebra II

Curriculum Guide 2017

Mission Statement

The mission of the Township of Union Public Schools is to build on the foundations of honesty, excellence, integrity, strong family, and community partnerships. We promote a supportive learning environment where every student is challenged, inspired, empowered, and respected as diverse learners. Through cultivation of students' intellectual curiosity, skills and knowledge, our students can achieve academically and socially, and contribute as responsible and productive citizens of our global community.

Philosophy Statement

The Township of Union Public School District, as a societal agency, reflects democratic ideals and concepts through its educational practices. It is the belief of the Board of Education that a primary function of the Township of Union Public School System is to formulate a learning climate conducive to the needs of all students in general, providing therein for individual differences. The school operates as a partner with the home and community.

Course Description

Algebra II is an extension of the Algebra 1 curriculum. Topics that were first introduced in Algebra 1 will be built upon and applied to problems that require higher order thinking skills. Additional topics will also be presented in a variety of methods, including self-discovery activities, group project and presentations, and teacher-led class discussions. Algebra 2 builds a foundation of mathematics for those students going on to Pre-Calculus and/or students who are college bound. Along with many colleges, a majority of careers require a successful completion of an Algebra 2 course. Fundamental skills of mathematics will be applied to such topics as functions, equations and inequalities, probability and statistics, logarithmic and exponential relationships, quadratic and polynomial equations, and matrices. Technology will be used to introduce and expand upon the areas of study listed above. Use of computers and graphing calculators will be incorporated into each unit.

Overview	Standards for Mathematical Content	Unit Focus	Standards for Mathematical Practice
Unit 1	N.CN.A.1 N.CN.A.2 N.CN.C.7 A.REI.B.4 A.REI.B.4b A.REI.C.7 A.REI.C.6 F.BF.A.2 F.LE.A.2 F.LE.B.5 A.SSE.B.4 A.APR.C.4 A.REI.D.11 F.BF.B.3 F.BF.A.1 F.BF.A.1b N.Q.A.2	• Perform arithmetic operations with complex numbers • Use complex numbers in polynomial identities and equations • Build a function that models a relationship between two quantities • Construct and compare linear, quadratic, and exponential models • Write expressions in equivalent forms to solve problems • Solve systems of equations in two and three variables • Solve a quadratic equation • Identify mathematical patterns • Use a formula to find the nth term • Find the sum of a series	MP.1 Make sense of problems and persevere in solving them. MP.2 Reason abstractly and quantitatively. MP.3 Construct viable arguments & critique the reasoning of others. MP.4 Model with mathematics. MP.5 Use appropriate tools strategically.
Unit 1: ***Suggested Educational Resources***	N.CN.A.1 Complex number patterns N.CN.A.2 Powers of a complex number N.CN.C.7, A.REI.B.4b Completing the square A.REI.C.7 Linear and Quadratic System A.REI.C.6 Pairs of Whole Numbers F.BF.A.2 Snake on a Plane F.LE.B.5, F.LE.A.2 Exponential Parameters A.SSE.B.4 Course of Antibiotics A.REI.D.11 Ideal Gas Law F.LE.A.2 Rumors F.BF.A.1b A Sum of Functions		

Unit 2	N.RN.A.1 N.RN.A.2 A.APR.B.2 A.SSE.A.2 F.IF.C.7 F.IF.C.7c A.APR.D.6 A.REI.A.2 A.REI.A.1 F.IF.B.4 F.IF.B.6 F.BF.B.3	• Construct and compare linear, quadratic, & exponential models • Extend the properties of exponents to rational exponents • Understand the relationship between zeros and factors of polynomials • Interpret the structure of expressions • Use polynomial identities to solve problems • Analyze functions using different representations • Rewrite rational expressions • Understand solving equations as a process of reasoning and explain the reasoning • Graph and analyze the graphs of polynomial equations • Divide polynomials • Simplify radical expressions • Solve radical equations • Solve rational equations	MP.6 Attend to precision. MP.7 Look for and make use of structure. MP.8 Look for and express regularity in repeated reasoning.
Unit 2: ***Suggested Educational Resources***	N.RN.A.1 Evaluating Exponential Expressions N.RN.A.2 Rational or Irrational? A.APR.B.2 The Missing Coefficient A.SSE.A.2 A Cubic Identity F.IF.C.7c Graphs of Power Functions A.APR.D.6 Combined Fuel Efficiency A.REI.A.1 Products and Reciprocals A.REI.A.2 Radical Equations A.REI.A.2, A.CED.A.1 An Extraneous Solution F.IF.B.4, F.IF.C.7e Model air plane acrobatics F.BF.B.3 Transforming the graph of a function		

<table>
<tr>
<td><u>Unit 3</u></td>
<td>A.SSE.B.3c
A.SSE.B.3
F.IF.C.8
F.IF.C.8b
F.LE.A.4
F.IF.C.7
F.TF.A.1
F.TF.A.2
F.IF.C.7e
F.TF.B.5
F.TF.C.8
F.BF.B.3</td>
<td>• Construct and compare linear, quadratic, and exponential models
• Write and evaluate logarithmic expressions
• Use the properties of logarithms
• Solve exponential and logarithmic equations
• Evaluate and simplify natural logarithmic expressions
• Solve equations using natural logarithms
• Extend the domain of trigonometric functions using the unit circle
• Analyze functions using different representations
• Interpret functions that arise in applications in the terms of the context
• Model periodic phenomena with trigonometric functions
• Prove and apply trigonometric identities
• Summarize, represent, and interpret data on two categorical and quantitative variables
• Build new functions from existing functions
• Build a function that models a relationship between two quantities</td>
<td rowspan="2">MP.1 Make sense of problems and persevere in solving them.

MP.2 Reason abstractly and quantitatively.

MP.3 Construct viable arguments & critique the reasoning of others.

MP.4 Model with mathematics.

MP.5 Use appropriate tools strategically.

MP.6 Attend to precision.</td>
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<tr>
<td>Unit 3:

Suggested Educational Resources</td>
<td colspan="2"><u>A.SSE.B.3c Forms of exponential expressions</u>

<u>F.IF.C.8b Carbon 14 dating in practice I</u>

<u>F.LE.A.4 Carbon 14 dating</u>

<u>F.IF.C.7e Logistic Growth Model</u>

<u>F.TF.A.1 Bicycle Wheel</u>

<u>F.TF.A.2 What exactly is a radian?</u>

<u>F.TF.A.2 Trigonometric functions for arbitrary angles (radians)</u>

<u>F.TF.A.2 Trig Functions and the Unit Circle</u>

<u>F.IF.B.4, F.IF.C.7e Model air plane acrobatics</u>

<u>F.TF.B.5 As the Wheel Turns</u>

<u>F.TF.C.8 Trigonometric Ratios and the Pythagorean Theorem</u></td>
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	F.BF.B.3 Exploring Sinusoidal Functions F.BF.B.3 Transforming the graph of a function		MP.7 Look for and make use of structure. MP.8 Look for and express regularity in repeated reasoning.
Unit 4	S.ID.A.4 S.IC.A.1 S.IC.A.2 S.IC.B.3 S.IC.B.4 S.IC.B.5 S.IC.B.6 S.CP.A.1 S.CP.A.2 S.CP.A.3 S.CP.A.4 S.CP.A.5 S.CP.B.6 S.CP.B.7	• Add, subtract, and multiply matrices • Solve matrix equations • Summarize, represent, and interpret data on a single count or measurement variable • Understand and evaluate random processes underlying statistical experiments • Make inferences and justify conclusions from sample surveys, experiments and observational studies • Understand the independence and conditional probability and use them to interpret data • Use the rules of probability to compute probabilities of compound events in a uniform probability model	
Unit 4: ***Suggested Educational Resources***	S.ID.A.4 Do You Fit in This Car? S.IC.A.1School Advisory Panel S.IC.A.2 Sarah, the chimpanzee S.IC.B.3 Strict Parents S.IC.B.4 Margin of Error for Estimating a Population Mean S.CP.A.1 Describing Events S.CP.A.2 Cards and Independence S.CP.A.3 Lucky Envelopes S.CP.A.4 Two-Way Tables and Probability S.CP.A.5 Breakfast Before School S.CP.B.6 The Titanic 1 S.CP.B.7 The Addition Rule S.CP.B.7 Rain and Lightning		

Unit 1 Algebra 2			
Content & Practice Standards	**Standards for Mathematical Practice**	**Critical Knowledge & Skills**	**Examples**
N.CN.A1. Know there is a complex number i such that i^2 = -1, and every complex number has the form a+bi with a and b real **N.CN.A.2** Use the relation i^2 = -1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	6, 7	Concepts: Complex number i is defined such that i squared =-1 Every complex number has the form a+bi with a and b real. Students are able to: i^2 and the commutative, associative properties to add and subtract complex numbers are to be used. determine that i^2 = -1 and the commutative, associative and distributive properties to multiply complex numbers Goal: Add, subtract, and multiply complex numbers using the commutative, associative and distributive properties.	Simplify the square root of -18 Simplify (2+i)+(-3+3i) Simplify (-6-2i)-(4+2i)
N.CN.C.7. Solve quadratic equations with real coefficients that have complex solutions A.REI.B.4. Solve quadratic equations in one variable. **A.REI.B.4b.** Solve quadratic equations by inspection, taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a + - bi for real numbers a and b.	5, 7	Concepts: As with real solutions, complex solutions to quadratic equations may be determined by taking square roots, factoring, and completing the square Students are able to: solve quadratic equations in one	Factor x^2 +3x +2 Complete the square x^2 +2x -33=0 A model rocket is launched with an initial velocity of 2000ft/s. The height in feet of the rocket after t seconds after the launch is given by h= -16t^2 + 200t. How many seconds after the launch will the rocket be 3300 ft above the ground?

		variable that have complex solutions by taking square roots solve a quadratic equation in one variable that has complex solutions by completing the square solve a quadratic equation in one variable that has complex solutions by factoring write complex solutions in a + - bi form Goal: Solve quadratic equations with real coefficients that have complex solutions by taking square roots, completing the square and factoring	
A.REI.C.7 Solve a simple system consisting of a linear equation and a quadratic equation and a quadratic equation in two variables algebraically and graphically.	1	Concepts: Solutions of linear systems contain different function types. Students are able to: solve a system containing one linear equation and one quadratic equation algebraically graph a system containing one linear equation and one quadratic equation to determine a solution Goal: Solve simple systems consisting of a linear and quadratic equation in two variables algebraically and graphically	Find the solution to the following system by substitution y-2x=-x^2-4 y+2x=-1 Find the solution to the following by elimination y=-x-7 y=x^2 -4x-5 Graph the following system and using the graphing calculator indicate the number of solutions and what the solutions are for the following system y=x^+1

			y=x+1 Functions f an g are defined as f(x)= 1/2x and g(x) =x^2 If the graphs of f and g intersect at P, find the x-coordinate of point P. Let f(x)=ax^2 where a >0, and g(x)=mx+b where m>0 and b<0. The equation f(x)=g(x) has n distinct solution(s). What are the possible values of n?
A.REI.C.6 Solve systems of linear equations exactly and approximately with graphs, focusing on pairs of linear equations in two variables.	1, 7	Concepts: Solving a system of linear equations containing n variables requires n equations. Students are able to: use the substitution method and/or elimination method to find the solution of a system containing 3 linear equations Goal: Solve algebraically a system of 3 linear equations	Solve the following system of 3 equations x+y+z=6 2x-y+3z=9 -x+2y+2z=9 Marina had 24,500 to invest. She divided the money into 3 accounts. At the end of the year, she made 1300 interest . The annual yield on each of the accounts was 4%, 5.5% and 6%. If the amount of money in the 4% account was 4 times the amount in the 5.5% account, how much had she placed in each account?
F.BF.A.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the 2 forms. **F.LE.A.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or 2 input-output pairs. **F.LE.B.5.** Interpret the parameters in a linear or	1, 2, 4, 6, 7, 8	Concepts: Recursion Students are able to: distinguish between recursive and explicit formulas represent geometric and arithmetic sequences	After knee surgery, your trainer tells you to return to your jogging program slowly. He suggests jogging for 12 minutes each day for the first week. Each week thereafter, he suggests that you increase that time by 6 minutes per day. How many weeks will it be before you are up to

exponential function in terms of a context.		recursively represent geometric and arithmetic sequences with explicit formulas translate between recursive form and explicit form of geometric and arithmetic sequences recognize explicit formula for geometric sequences as exponential functions containing a domain in the integers only interpret the parameters of an exponential function representing a geometric sequence interpret the parameters of a linear function representing an arithmetic sequence Goal: Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.	jogging 60 minutes per day? A culture of bacteria doubles every 2 hours. If there are 500 bacteria at the beginning, how many bacteria will there be after 24 hours?
A.SSE.B.4 Derive and/or explain the derivation of the formula for the sum of a finite geometric series and use the formula to solve problems.	1, 7	Concepts: Series as a sum of a sequence Students are able to: derive or explain the derivation of the formula for the sum of a finite geometric series use the formula for the sum of a finite geometric series to solve problems Goal:	Calculate a mortgage payment.

		Use the formula for the sum of a finite geometric series to solve problems like mortgage payments.	
A.APR.C.4 Prove polynomial identities and use them to describe numerical relationships.	3, 7	Concepts: Polynomial identities can be used to describe numerical relationships. show the polynomial identity which can be used to generate Pythagorean triples. prove polynomial identities Goal: Use polynomial identities to describe numerical relationships and prove polynomial identities.	Generate a pythagorean triple using (x^2+y^2)^2=(x^2-y^2)^2+(2xy)^2
A.REI.D.11 Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y=g(x) intersect are the solutions of the equation f(x)=g(x); find the solutions approximately using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.	1, 5	Concepts: Solutions to complex systems of nonlinear functions can be approximated graphically Students are able to: find the solution to f(x)=g(x) using graphing technology in cases of linear, polynomial, rational, absolute value, exponential and logarithmic functions find the solution to f(x)=g(x) by making tables of values or find successive approximations in linear, polynomial, rational, absolute value, exponential and logarithmic cases. Goal:	Use a graphing calculator to solve f(x)=g(x), where f(x)= 3x^2 +5x -3 and g(x)= log x

		Find approximate solutions for f(x)=g(x) using technology to graph, make tables of values, or find successive approximations. Include cases where the functions are linear, polynomial, rational, absolute value, logarithmic and exponential.	
F.BF.B3. Identify the effect on the graph of f(x) by replacing with f(x)+k, kf(x),f(kx) and f(x+k) for specific values of k; find the value of k given the graphs. Experiment with cases and illustrate an explain effects on the graph using technology. Include even and odd functions.	3, 5, 7, 8	Concepts: Function notation representations of transformations Students will be able to: perform transformations on graphs of polynomial, exponential, logarithmic or trigonometric functions identify horizontal and vertical shifts and horizontal and vertical stretches and shrinks identify the effect on the graph of combinations of transformations given the graph find the value of the constant that transformed it illustrate an explanation of the effects on polynomial, exponential, logarithmic or trigonometric graphs using technology Goal: identify the effect on the graph of an exponential, polynomial, logarithmic or trigonometric function by replacing f(x) with f(x)=k, kf(x), f(x+k) or f(kx) with positive and negative values of	Let f(x)= x^2. Describe and graph the transformations, f(x+5), f(x)-9, 8f(x) and f(3x). Use a graphing calculator to check your results. If g(x)= log x, write a new equation that represents a transformation of the graph of g(x) 3 down and 7 to the right.

		k. Find the value of k given graphs and identify even and odd functions from graphs and equations	
F.BF.A.1 Write a function that describes a relationship between 2 quantities. **F.BF.A.1b**. Combine standard function types using arithmetic operations. **N.Q.A.2** Define appropriate quantities for the purpose of descriptive modeling	4, 7	Concepts: Functions of various types can be combined to model real world situations. Students will be able to: use arithmetic operations to combine functions of varying types in order to model realities Goal: Construct a function that combines, using arithmetic operations, standard function types to model a relationship between 2 quantities	Build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

Unit 1 Vocabulary		
dependent system equivalent systems independent system linear system system of equations solution of a system of equations axis of symmetry complex number discriminant	parabola Quadratic Formula quadratic function standard form vertex form zero of a function arithmetic sequence arithmetic series common difference	explicit formula geometric sequence geometric series limits recursive formula imaginary number converge diverge greatest common factor common ratio

Unit 2 Algebra 2			
Content & Practice Standards	**Standards for Mathematical**	**Critical Knowledge & Skills**	**Examples**

	Practice		
A.APR.B.2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.	6	Concepts: · Polynomial division: For a polynomial $p(x)$ and a number a: - $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$ - $(x - a)$ is a factor of $p(x)$ if and only if $p(a) =$ Students are able to use the Remainder Theorem to determine factors of a polynomial. Learning Goal 1: Apply the Remainder Theorem in order to determine the factors of a polynomial.	1) Use the Remainder Theorem to determine if $x + 1$ is a factor of $x^3 - x^2 - 2x$. 2) You want to buy carpeting for a rectangular room. If the area of the room is $x^2 - 13x + 36$, what could be possible binomial dimensions of the room?
A.SSE.A.2. Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$. **A.APR.B.3.** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	7	Concepts: · Factors of polynomials can be used to identify zeros to be used to develop a rough graph of the polynomial function. Students are able to: · factor polynomials. · analyze a table of values to determine where the polynomial is increasing and decreasing. · use the zeros of the polynomial to create rough graph. Learning Goal 2: Use an appropriate factoring technique to factor polynomials. Explain the relationship between zeros and factors of polynomials, and use the zeros to construct a rough graph of the function defined by the polynomial.	Write an equation for a polynomial function with three turning points and end behavior up and up.
F.IF.C.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. **F.IF.C.7c.** Graph polynomial	1, 5, 6	Concepts: · Factors of polynomials can be used to identify zeros to be used to develop a rough graph of the polynomial function. Students are able to: · graph a polynomial function given its equation. · identify zeros from the graph and using an appropriate factoring technique. · show key features of the graph, including end behavior.	Write a polynomial function with 3 zeros, one zero of 1 and another zero with multiplicity of 2.

functions, identifying zeros when suitable factorizations are available, and showing end behavior.		· use technology to graph and describe key features of the graph for complicated cases. Learning Goal 3: Graph polynomial functions from equations; identify zeros when suitable factorizations are available; show key features and end behavior.	
A.APR.C.4. Prove polynomial identities and use them to describe numerical relationships. *For example, the difference of two squares; the sum and difference of two cubes; the polynomial identity* $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.*	3, 7	Concepts: · Polynomial identities can be used to describe numerical relationships. Students are able to: · show that the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples. · prove polynomial identities. Learning Goal 4: Use polynomial identities to describe numerical relationships and prove polynomial identities.	Check the polynomial identity $(x+y)^2 = x^2 + 2xy + y^2$ using $x = 6$ and $y = 3$
A.APR.D.6. Rewrite simple rational expressions in different forms; write *a*(*x*)/*b*(*x*) in the form *q*(*x*) + *r*(*x*)/*b*(*x*), where *a*(*x*), *b*(*x*), *q*(*x*), and *r*(*x*) are polynomials with the degree of *r*(*x*) less than the degree of *b*(*x*), using inspection, long division, or, for the more complicated examples, a computer algebra system.	1	Concepts: · Rational expressions can be written in different forms. Students are able to: · write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x). · use inspection, factoring and long division to rewrite rational expressions. · use technology to rewrite rational expressions for more complicated cases. Learning Goal 5: Rewrite simple rational expressions in different forms using inspection, long division, or, for the more complicated examples, a computer algebra system.	1) Using long division, simplify $\frac{12x^2 + 11x - 5}{3x - 1}$ 2) You are rolling out the crust for a rectangular deep dish pizza. The area of the pie is $6x^2 + x - 1$ and the length is $2x + 1$. How wide should the crust be?
A.REI.A.2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. **A.REI.A.1.** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous	2, 3, 4, 6	Concepts: · Inverse relationships exist between roots and powers. · Extraneous solutions do not result in true statements. Students are able to: · use the inverse relationship between roots and powers when solving radical equations. · identify any extraneous solutions. · solve simple rational equations in one variable (degree of numerators and denominator is not greater than 2). · write simple rational equations in one variable and use the	1) Error Analysis. A student said that 4 and 1 are the solutions of the following radical equation. Describe and correct the student's error. $\sqrt{x} + 2 = x$ 2) Write a simplified expression for the area of the rectangle whose length is $\frac{3a+9}{2a-6}$ and width is $\frac{4a+4}{a+3}$. State all restrictions

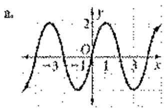
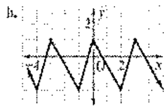
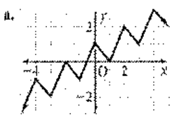
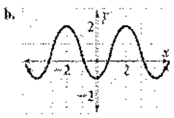
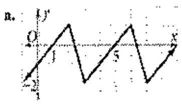
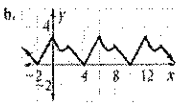
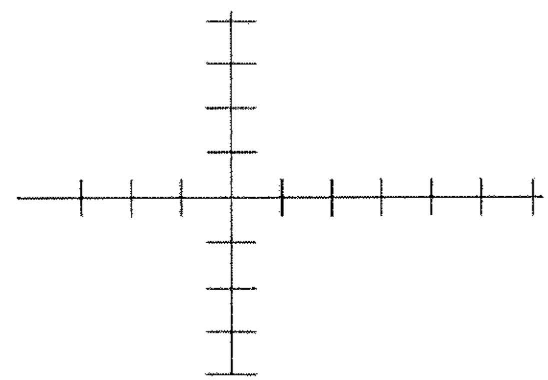
step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. **A.CED.A.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.		rational equation to solve problems. Learning Goal 6: Solve simple rational and radical equations in one variable, use them to solve problems and show how extraneous solutions may arise. Create simple rational equations in one variable and use them to solve problems.	on a. 3) Open-Ended. Explain how factoring is used when adding or subtracting rational expressions. Include an example in your explanation.
F.IF.B.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. **F.IF.B.6.** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.	1, 4, 5, 6, 7	Concepts: · A radical function is any function that contains a variable inside a root. Students are able to: · interpret key features of radical functions from graphs and tables in the context of the problem. · sketch graphs of radical functions given a verbal description of the relationship between the quantities. · identify intercepts and intervals where function is increasing/decreasing. · determine the practical domain of a radical function. · determine key features including intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maxima and minima; symmetries; end behavior. Learning Goal 7: For radical functions, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.	1)A sprinkler can water between 1 and 130 square yards of a lawn. The length L in inches of rotating pipe needed to water A square yards is given by the function $L = 117.75\sqrt{A}$. a. Graph the equation on your calculator. Make a sketch of the graph. b. How much area can be watered if the length of the pipe is 500,800, or 1,300 inches long? 2) Graph $y = \sqrt{-x}$, $y = \sqrt{1-x}$ and $y = \sqrt{2-x}$. How does the graph of $y = \sqrt{h-x}$ differ from the graph of $y = \sqrt{x-h}$?
F.IF.C.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. F.IF.C.7e. Graph exponential and	1, 5	Concepts: · Logarithmic functions Students are able to: · graph logarithmic functions having base 2, 10 or e, using technology for more complicated cases. · show intercepts and end behavior of logarithmic functions.	The population of a certain animal species decreases at a rate of 3.5% per year. You have counted 80 of the animals in the habitat you are studying. a. Write a function that models the

logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.		Learning Goal 9: Graph logarithmic functions expressed symbolically and show key features of the graph (including intercepts and end behavior).	change in the animal population. b. Graph the function. Estimate the number of years until the population first drops below 15 animals.
A.REI.D.11. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*	1, 5	Concepts: · Solutions to complex systems of nonlinear functions can be approximated graphically Students are able to: · find the solution to f(x)=g(x) approximately, e.g., using technology to graph the functions; include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. · find the solution to f(x)=g(x) approximately, e.g., using technology to make tables of values, or find successive approximations; include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. Learning Goal 10: Find approximate solutions for f(x)=g(x), using technology to graph, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, logarithmic and exponential functions.	Use a graphing calculator, or graphing software, to approximate the solutions to the following systems: a. $f(x) = \begin{cases} x^2 - y = 0 \\ x + y = 2 \end{cases}$ b. $f(x) = \begin{cases} y = \|x\| \\ y = x^2 \end{cases}$ c. $f(x) = \begin{cases} y = log(x+5) \\ y = x^2 \end{cases}$

Unit 2 Vocabulary		
Polynomial, monomial, binomial, trinomial, factor, remainder, multiply, divide, divisor, dividend, quotient, length, width, perimeter, area, factor theorem, multiple zero, multiplicity, relative maximum,	monomial, degree of a polynomial, polynomial function, standard form of a polynomial, turning point, end behavior	relative minimum, polynomial identity, difference of two squares, sum and difference of two cubes, radical equation, square root equation, rational expression, simplest form, complex fraction, radical functions, square root functions

Unit 3 Algebra 2			
Content & Practice Standards	**Standards for Mathematical**	**Critical Knowledge & Skills**	**Examples**

	Practice		
F.TF.A.1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. **F.TF.A.2**. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	3, 6	**Concepts**: · Radian measure of an angle as the length of the arc on the unit circle that is subtended by the angle · Relationship between degrees and radians **Students are able to**: · find the measure of the angle given the length of the arc. · find the length of an arc given the measure of the central angle. · convert between radians and degrees. · use the unit circle to evaluate sine, cosine and tangent of standard reference angles. **Goals**: Use the radian measure of an angle to find the length of the arc in the unit circle subtended by the angle and find the measure of the angle given the length of the arc. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	**When an object travels on a circular path, its angular velocity, ω, is the rate at which Θ changes. Angular velocity is defined by the equation, where Θ is usually expressed in radians and *t* represents times. Find the angular velocity in radians per second of a point on a bicycle tire if it completes two revolutions in 3 seconds.Earth rotates on its axis once every 24 hours.** **a. How long does it take Earth to rotate through an angle of 300°?** **b.How long does it take Earth to rotate through an angle of radians?**

F.IF.C.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. **F.IF.C.7e**. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	**1,4,5,6,7**	**Concepts:** · Relationship between the unit circle in the coordinate plane and graph of trigonometric functions. **Students are able to:** · graph trigonometric functions, showing period, midline, and amplitude. **Goal**: Graph trigonometric functions expressed symbolically, showing key features of the graph, by hand in simple cases and using technology for more complicated cases.	Example 1: For each function, identify one cycle in two different ways. Then determine the period of the function. a.  b.  Example 2: Determine whether each function is or is not periodic. If it is, find the period. a.  b.  Example 3: Find the amplitude of each function. a.  b. 
F.TF.B.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.	4	**Concepts:** · Periodic functions may model real-world scenarios. Students are able to: · use characteristics of real world phenomena to select a trigonometric model. · identify amplitude, frequency and midline appropriate for the model. **Goal** : Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.	**2.** $y = 4\sin \pi x$ Period: ________ Amplitude:________ Reflection:________ Phase Shift:________ Vertical Shift:________ Transformations: 

<table>
<tr><td></td><td></td><td></td><td>Write the equation of a cosine function of the form $y = A\cos(Bx - C) + D$ that has the given characteristics.
9. Amplitude: 2
Period: $\frac{\pi}{3}$
Shift: Down 1 unit

Ferris Wheel
1. As you ride the Ferris wheel, your distance from the ground varies sinusoidally with time. You are the last seat filled at the bottom and the Ferris wheel starts immediately. Let t be the number of seconds that have elapsed since the Ferris wheel started. You find that is takes you 4 seconds to reach the top, 43 feet above the ground, and that the wheel makes a revolution once every 8 seconds. The diameter of the wheel is 40 feet.
1. Sketch a graph
2. What is the lowest you go as the Ferris wheel turns, and why is this number greater than zero?
3. Write an equation.
4. Predict your height above the ground when:
i. t=6
ii. t =
iii. t =0

What is the value of the second time you are 18 feet above the ground?</td></tr>
<tr><td>F.TF.C.8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find sin(θ), cos(θ),or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle.</td><td>3,5,7</td><td>Students are able to:
· prove the Pythagorean identity: $\sin^2(\theta) + \cos^2(\theta) = 1$.
· use the Pythagorean identity to find sin(θ), cos(θ), or tan(θ) when given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle.
Goal: Use the Pythagorean identity $(\sin \theta)^2 + (\cos \theta)^2 = 1$ to</td><td>Represent the number 1 using only 2 trigonometric functions.</td></tr>
</table>

		find sin θ , cos θ , or tan θ , given sin θ , cos θ , or tan θ , and the quadrant of the angle.	
F.BF.B.3. Identify the effect on the graph of replacing *f*(*x*) by *f*(*x*) + *k*, *k f*(*x*), *f*(*kx*), and *f*(*x* + *k*) for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*	**3,5,7,8**	**Concepts**: · Function notation representation of transformations Students are able to: · perform transformations on graphs of polynomial, exponential, logarithmic, or trigonometric functions. · identify the effect on the graph of replacing f(x) by - *f*(*x*) + *k;* - *k f*(*x*); - *f*(*kx*); - *and f*(*x* + *k*) for specific values of *k* (both positive and negative). · identify the effect on the graph of combinations of transformations. · given the graph, find the value of k. · illustrate an explanation of the effects on polynomial, exponential, logarithmic, or trigonometric graphs using technology. **Goal** : Identify the effect on the graph of a polynomial, exponential, logarithmic, or trigonometric function of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative). Find the value of k given the graphs and identify even and	

		odd functions from graphs and equations.	
F.LE.A.4. Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology.	2,4	**Concepts**: · Exponents and logarithms have an inverse relationship. · Solutions to an exponential equation in one variable can be written as a logarithm. Students are able to: transform an exponential model represented by $ab^{ct} = d$ where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*. write the solution to $ab^{ct} = d$ as a logarithm. use technology to evaluate logarithms having base 2, 10, or e. **Goal** : Express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.	
A.SSE.B.3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression **A.SSE.B.3c:** Use the properties of exponents to transform expressions for exponential functions. *For example the expression* 1.15^t *can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal*	**1, 2, 4, 7**	**Concepts**: · Alternate, equivalent forms of an exponential expression containing rational exponents may reveal specific attributes of the function that it defines. **Students are able to**: · use properties of exponent transform/rewrite an exponential expression for an exponential function.	**1) If you invest $500 in a savings account with a 1.2% annual interest rate, when will the account contain at least $650?** **2) Your friend says that the graph of** $f(x) = (\frac{3}{4})^{x+2} + 1$ **is a shift of the parent function two units up and one unit left. Describe and correct your friend's error.** **3) Graph** $y = \log_4 x$**. Describe the domain, range, y-intercept and asymptotes.**

the approximate equivalent monthly interest rate if the annual rate is 15%. **F.IF.C.8.** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function **F.IF.C.8b**: Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay.*		· explain the properties of the quantity or the function. **Goal** : Use the properties of exponents to transform expressions for exponential functions, explain properties of the quantity revealed in the transformed expression or different properties of the function.	

Unit 3 Vocabulary		
cycle period periodic function phase shift radian sine tangent	asymptote amplitude Change of Base Formula common logarithm exponential equation exponential function cosine	central angle exponential growth logarithm logarithmic equation logarithmic function natural logarithmic function unit circle

Unit 4 Algebra 2			
Content & Practice Standards	**Standards for Mathematical Practice**	**Critical Knowledge & Skills**	**Examples**
S.ID.A.4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.	**2,4**	**Concepts**: · Mean and standard deviation are used to fit in a normal distribution · Population percentages may be estimated when the data are approximately normally distributed. Students are able to: · identify data sets as approximately normally distributed or not. · explain the 68-95-99.7 rule for normal distributions (approximately 68% of the area under a normal distribution curve is within one standard deviation, approximately 95% of the area under a normal distribution curve is within two standard deviations, etc). · use the mean and standard deviation of a normal distribution to estimate population percentages. · use calculators, spreadsheets, and tables to estimate areas under the normal curve and interpret in context. **Goal**: Use the mean and standard deviation of a data set to fit it to a normal distribution, estimate population percentages, and recognize that there are data sets for which such a procedure is not appropriate (use calculators, spreadsheets,	**Identify data sets as approximately normally distributed or not.**

		and tables to estimate areas under the normal curve).	

S.IC.A.1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.	2,4	**Concepts**: · Statistics is a process for making inferences about a population based on analysis of a random sample from the population. Students are able to: · identify and evaluate random sampling methods. · explain the importance of randomness to sampling and inference making. · explain the difference between values that describe a population and a sample, in context. **Goal**: Identify and evaluate random sampling methods.	**1. Analyzing Sampling Methods:** **Public Opinion - A newspaper wants to find out what percent of a city population favors a property tax increase to raise money for local parks. What is the sampling method used for each situation? Does the sample have a bias? Explain.** **a) A newspaper article on the tax increase invites readers to call the paper and express their opinions** **b) A reporter interviews people leaving the city's largest park** **c) A survey service calls every 50th listing from the local phone book** **2. Identify the sampling method then identify any bias in each method:** **a) A supermarket wants to fin the percent of shoppers who use coupons. A manager interviews every shopper entering the greeting card aisle.** **b) A maintenance crew wants to estimate how many of 3,000 air filters in an office building need replacing. The crew examines five filters chosen at random on each floor of the building.** **c) A student government wants to find out how many students have after-school jobs. A pollster interviews students selected at random as they board busses at the end of the school day.**

S.IC.A.2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*	2,4	**Concepts**: · Random processes can be described mathematically by using a model: a list or description of possible outcomes. Students are able to: · determine whether a given model is consistent with results from and experiment. · know the difference between experimental and theoretical modeling. · know how far predictions can be projected based on sample size. · design simulations of random sampling. **Goal**: Determine if the outcomes and properties of a specified model are consistent with results from a given data-generating process (e.g. using simulation).	**A model says a spinning coin lands heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?**
S.IC.B.3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.	4	**Concepts**: · Collecting data from a random sample of a population makes it possible to draw conclusions about the whole population. · Randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. · Sample surveys, experiments, and observational studies serve different statistical purposes allowing for different statistical analyses. **Students are able to**: · distinguish between sample surveys, experiments, and observational studies. · explain the importance of randomization in each of these processes.	**1. Suppose you are conducting a survey about careers, write a survey question using each of the following biases: a) leads people to a particular response b) does not provide enough information c) combines two or more issues d) is too wordy or confusing 2. a) Data Collection: Write a survey question to find out the number of students at your school who plan to continue their education after high school b) Describe the sampling method you would use**

		· identify voluntary response samples and convenience samples. · describe simple random samples, stratified random samples, and cluster samples. · explain how undercoverage, nonresponse, and question wording can lead to bias in a sample survey. **Goal**: Identify the differences among and purposes of sample surveys, experiments, and observational studies, explaining how randomization relates to each.	**c) Conduct your survey**
S.IC.B.4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling	**1,2,4,5,6**	**Concepts**: · Appropriately drawn samples of a population may be used to estimate a population mean or population proportion. · Relationship between margin of error, variation with a data set, and variability in the population Students are able to: · conduct simulations of random sampling to gather samples. · estimate population means with sample means. · estimate population proportions with sample proportions. · calculate margins of error for the estimates. · explain how the results relate to variability in the population. **Goal**: Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.	**1. Explain the differences between measures of central tendency and measures of variation.** **2. Compare & Contrast: Three data sets each have a mean of 70. Set A has a standard deviation of 10. Set B has a standard deviation of 5. Set C has a standard deviation of 20. Compare and contrast these three sets.** **3. Approximate the margin of error for the sample. In 2007, the US Mint began issuing $1 coins featuring images of the nation's presidents. In a survey, 76% of 2431 US adults opposed using $1 coins.**

<table>
<tr>
<td>S.IC.B.5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant
S.IC.B.6. Evaluate reports based on data.</td>
<td>1,2,4,5,6</td>
<td>Concepts:
· A statistically significant outcome is one that is unlikely to be due to chance alone.
Students are able to:
· conduct a t-test to evaluate the effectiveness and differences in two treatments.
· use simulations to generate data simulating applying two treatments.
· use the results of simulations to determine if the differences are significant.
· read and explain, in the context of the situation, data from outside reports – discussing experimental study design, drawing conclusions from graphical and numerical summaries, and identifying characteristics of the experimental design.

Goal: Use data from a randomized experiment to compare two treatments and use simulations to decide if differences between parameters are significant; evaluate reports based on data.</td>
<td>1. Sam Sleep researcher hypothesizes that people who are allowed to sleep for only four hours will score significantly lower than people who are allowed to sleep for eight hours on a cognitive skills test. He brings sixteen participants into his sleep lab and randomly assigns them to one of two groups. In one group he has participants sleep for eight hours and in the other group he has them sleep for four. The next morning he administers the SCAT (Sam's Cognitive Ability Test) to all participants. (Scores on the SCAT range from 1-9 with high scores representing better performance).
<table>
<tr><th colspan="9">SCAT Scores</th></tr>
<tr><td>8 hours</td><td>5</td><td>7</td><td>5</td><td>3</td><td>5</td><td>3</td><td>3</td><td>9</td></tr>
<tr><td>4 hours</td><td>8</td><td>1</td><td>4</td><td>6</td><td>6</td><td>4</td><td>1</td><td>2</td></tr>
</table></td>
</tr>
<tr>
<td>S.CP.A.1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not")</td>
<td>1,2,4,5,6</td>
<td>Concepts:
· Events are described as subsets of a sample space.

Students are able to:
· identify a sample space, recognizing it as the set of all possible outcomes.
· identify and describe subsets of a sample space as events.
· describe unions, intersections and complements of events.
· visualize unions, intersections and complements of events with Venn diagrams.</td>
<td>A multiple choice test has four choices for each answer. Suppose you make a random guess on three of the ten test questions. What is the probability you will answer all three correctly?
Two standard number cubes are rolled. What is the probability that the sum is greater than 9 or less than 6?</td>
</tr>
</table>

		Goal: Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").	
S.CP.A.2. Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent. **S.CP.A.3**. Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*. **S.CP.A.4**. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their*	**1,2,4,5,6**	**Concepts:** · Two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities. · Independence of event *A* and event *B* means that the conditional probability of *A* given *B* is the same as the probability of, and the conditional probability of *B* given *A* is the same as the probability of *B*. **Students are able to:** · identify events as independent or dependent. · interpret the conditional probability of A given B as answering the question 'now that B *has* occurred, what is the probability that event A will occur?'. · determine the conditional probability of *A* given *B* using *P*(*A* and *B*)/*P*(*B*). · represent conditional probability of *A* given *B* as P(A|B). · calculate conditional probabilities. · construct two-way frequency tables for two categorical variables. · calculate probabilities from the two-way frequency table. · use the probabilities to assess independence of two variables. **Goal**: Use two-way frequency tables to determine if events are independent and to calculate conditional probability. Use everyday language to explain independence and conditional	***After collecting data from students on their favorite subject, estimate the probability a student will favor science GIVEN they are in the tenth grade.*** ***Compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.***

favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results. **S.CP.A.5.** Recognize and explain the NEW Concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*		probability in real-world situations.	
S.CP.B.6. Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model. **S.CP.B.7**. Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.	**1,2,4,5,6**	**Concepts**: · Mutually exclusive events exist. Students are able to: · analyze event B's outcomes to determine the proportion of B's outcomes that also belong to event A. · interpret this proportion as conditional probability of A given B. · identify two events as mutually exclusive (disjoint). · calculate probabilities using the Addition rule P(A or B) = P(A) + P(B) – P(A and B). **Goal**: Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A* and apply the Addition Rule [*P(A* or *B)* =	**Determine the probability of the Yankees and Mets both winning on a day they are both playing other teams.**

		$P(A) + P(B) - P(A \text{ and } B)$].	

Unit 4 Vocabulary	
theoretical probability combination conditional probability experimental probability measure of central tendency mutually exclusive events normal distribution permutation simulation	sample survey standard deviance variance determinant dilation equal matrices matrix equation scalar multiplication

Research-Based Effective Teaching Strategies	**21st Century Learning Skills**
-Task/Activities that solidifies mathematical concepts -Use questioning techniques to facilitate learning -Reinforcing Effort, Providing Recognition -Practice , reinforce and connect to other ideas within mathematics -Promotes linguistic and nonlinguistic representations -Cooperative Learning Setting Objectives, Providing Feedback -Varied opportunities for students to communicate mathematically -Use technological and /or physical tools	-Teamwork and Collaboration Initiative and Leadership Curiosity and Imagination -Innovation and Creativity -Critical thinking and Problem Solving -Flexibility and Adaptability -Effective Oral and Written Communication -Accessing and Analyzing Information

Formative Assessment	**Summative Assessment**	**Technology**

Short constructed responses Extended responses Checks for understanding Exit tickets Teacher observation Projects Timed Practice Test – Multiple Choice & Open-Ended Questions	Summer Packet Test 2 Variable Systems Quiz 3 Variable Systems Quiz 9.2-9.5 Test Quadratic Quiz Solving Quadratic Quiz Quadratics Test Complex Number Quiz MP1 Benchmark Polynomials Quiz Polynomials Test Rational Expressions Quiz Rational Expressions/Equations Test Radicals Quiz Radicals Test Midterm Exam Graphing Radicals Quiz Growth/Decay Quiz Logs Quiz Logs Test Unit Circle/Angles Quiz Graphing Functions Test Trig Quiz MP3 Benchmark Probability Test Statistics Test Matrices Test Final Exam	NJ CORE Annenberg Learning : Insight into Algebra 1 Mathematics Assessment Projects Get the Math Achieve the Core Webmath.com sosmath.com Mathplanet.com Interactive Mathematics.com Illustrative Mathematics Inside Mathmatics.org Asia Pacific Economic Cooperation : Lesson Study Videos Genderchip.org Interactive Geometry Mathematical Association of America National Council of Teachers of