TOWNSHIP OF UNION PUBLIC SCHOOLS

Grade 7 Mathematics
Curriculum Guide 2017

Mission Statement

The mission of the Township of Union Public Schools is to build on the foundations of honesty, excellence, integrity, strong family, and community partnerships. We promote a supportive learning environment where every student is challenged, inspired, empowered, and respected as diverse learners. Through cultivation of students' intellectual curiosity, skills and knowledge, our students can achieve academically and socially, and contribute as responsible and productive citizens of our global community.

Philosophy Statement

The Township of Union Public School District, as a societal agency, reflects democratic ideals and concepts through its educational practices. It is the belief of the Board of Education that a primary function of the Township of Union Public School System is to formulate a learning climate conducive to the needs of all students in general, providing therein for individual differences. The school operates as a partner with the home and community.

Course Description

Seventh grade mathematics is about (1) developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume; and (4) drawing inferences about populations based on samples.

Key Areas of Focus for Grade 7: Ratios and proportional reasoning; arithmetic of rational numbers.

Rationale for Module Sequence in Grade 7

In Module 1, students build on their Grade 6 experiences with ratios, unit rates, and fraction division to analyze proportional relationships. They decide whether two quantities are in a proportional relationship, identify constants of proportionality, and represent the relationship by equations. These skills are then applied to real-world problems including scale drawings. Students continue to build an understanding of the number line in Module 2 from their work in Grade 6. They learn to add, subtract, multiply, and divide rational numbers. Module 2 includes rational numbers as they appear in expressions and equations—work that is continued in Module 3. Module 3 consolidates and expands students' previous work with generating equivalent expressions and solving equations. Students solve real-life and mathematical problems using numerical and algebraic expressions and equations. Their work with expressions and equations is applied to finding unknown angles and problems involving area, volume, and surface area. Module 4 parallels Module 1's coverage of ratio and proportion, but this time with a concentration on percent. Problems in this module include simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, and percent error. Additionally, this module includes percent problems about populations, which prepare students for probability models about populations covered in the next module. In Module 5, students learn to draw inferences about populations based on random samples. Through the study of chance processes, students learn to develop, use and evaluate probability models. The year concludes with students drawing and constructing geometrical figures in Module 6. They also revisit unknown angle, area, volume, and surface area problems, which now include problems involving percentages of areas or volumes.

Recommended Textbook:

Eureka Math – EngageNY Grade 7 Mathematics

Overview	Standards for Mathematical Content	Unit Focus	Standards for Mathematical Practice
Unit 1 Ratios and Proportional Relationships (30 days) **Proportional relationships are also covered in Unit 4.**	**Analyze proportional relationships and use them to solve real-world and mathematical problems.14** **7.RP.1** Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute* 7.RP.2 Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. For example, if total cost t is	**Grade 7 Mathematics Module 1: Ratios and Proportional Relationship** In this 30-day Grade 7 module, students build upon sixth grade reasoning of ratios and rates to formally define proportional relationships and the constant of proportionality. Students explore multiple representations of proportional relationships by looking at tables, graphs, equations, and verbal descriptions. Students extend their understanding about ratios and proportional relationships to compute unit rates for ratios and rates specified by rational numbers. The module concludes with students applying proportional reasoning to identify scale factor and create a scale drawing. The student materials consist of the student pages for each lesson in Module 1. The copy ready materials are a collection of the module assessments, lesson exit tickets and fluency exercises from the teacher materials. • In Topic A, students examine	**Unit 1:** **MP.1 Make sense of problems and persevere in solving them.** Students make sense of and solve multi-step ratio problems, including cases with pairs of rational number entries; they use representations, such as ratio tables, the coordinate plane, and equations, and relate these representations to each other and to the context of the problem. Students depict the meaning of constant of proportionality in proportional relationships, the importance of $(0,0)$ and $(1,rr)$ on graphs, and the implications of how scale factors magnify or shrink actual lengths of figures on a scale drawing. **MP.2 Reason abstractly and quantitatively.** Students compute unit rates for paired data given in tables to determine if the data represents a proportional relationship. Use of concrete numbers will be analyzed to create and implement equations, including $yy=kkkk$, where kk is the constant of proportionality. Students decontextualize a given constant speed situation, representing symbolically the quantities involved with the formula, distance=rate×time. In scale drawings, scale factors will be changed to create additional scale drawings of a given picture. **Unit 2:** **MP.1 Make sense of problems and persevere in solving them.** When problem-solving, students use a variety of techniques to make sense of a situation involving rational numbers. For

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	proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn. d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate. 7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error. Solve real-life and mathematical problems using numerical and algebraic expressions and equations.15 7.EE.416 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form px + q = r and p(x + q) = r, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the	situations carefully to determine if they are describing a proportional relationship. Their analysis is applied to relationships given in tables, graphs, and verbal descriptions (**7.RP.2a**). • In Topic B, students learn that the unit rate of a collection of equivalent ratios is called the *constant of proportionality* and can be used to represent proportional relationships with equations of the form $y = kx$, where k is the constant of proportionality (**7.RP.2b**, **7.RP.2c**, **7.EE.4a**). Students relate the equation of a proportional relationship to ratio tables and to graphs and interpret the points on the graph within the context of the situation (**7.RP.2d**). • In Topic B, students learn that the unit rate of a collection of equivalent ratios is called the *constant of proportionality* and can be used to represent proportional relationships with equations of the form $y = kx$, where k is the constant of proportionality (**7.RP.2b**, **7.RP.2c**, **7.EE.4a**). Students relate the equation of a proportional relationship to ratio tables and to graphs and interpret the points on the graph within the context of the situation (**7.RP.2d**). • In Topic D, students bring the sum of their experience with proportional relationships to the context of scale drawings (**7.RP.2b**,	example, they may draw a number line and use arrows to model and make sense of an integer addition or subtraction problem. Or when converting between forms of rational numbers, students persevere in carrying out the long division algorithm to determine a decimal's repeat pattern. A tape diagram may be constructed as an entry point to make sense of a working-backward problem. As students fluently solve word problems using algebraic equations and inverse operations, they consider their steps and determine whether or not they make sense in relationship to the arithmetic reasoning that served as their foundation in earlier grades. **MP.2 Reason abstractly and quantitatively**. Students make sense of integer addition and subtraction through the use of an integer card game and diagramming the distances and directions on the number line. They use different properties of operations to add, subtract, multiply, and divide rational numbers, applying the properties to generate equivalent expressions or explain a rule. Students use integer subtraction and absolute value to justify the distance between two numbers on the number line. Algebraic expressions and equations are created to represent relationships. Students know how to use the properties of operations to solve equations. They make *zeros and ones* when solving an algebraic equation, thereby demonstrating an understanding of how the use of inverse operations ultimately leads to the value of the variable. **MP.4 Model with mathematics**. Through the use of number lines, tape diagrams, expressions, and equations, students model relationships between rational numbers. Students relate operations involving integers to contextual examples. For instance, an overdraft fee of \$25 that is applied to an account balance of −\$73.06 is represented by the expression −73.06−25 or −73.06+(−25) using the additive inverse. Students compare their answers and thought processes in the Integer Game and use number line diagrams to ensure accurate reasoning. They deconstruct a difficult word problem by writing an equation, drawing a number line, or drawing a tape diagram to represent quantities. To find a change in

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	perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width? Draw, construct, and describe geometrical figures and describe the relationships between them.17 7.G.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.	**7.G.1**). Given a scale drawing, students rely on their background in working with side lengths and areas of polygons (**6.G.1, 6.G.3**) as they identify the scale factor as the constant of proportionality, calculate the actual lengths and areas of objects in the drawing, and create their own scale drawings of a two-dimensional view of a room or building. The topic culminates with a two-day experience of students creating a new scale drawing by changing the scale of an existing drawing.	elevation, students may draw a picture representing the objects and label their heights to aid in their understanding of the mathematical operation(s) that must be performed. **MP.6 Attend to precision**. In performing operations with rational numbers, students understand that the decimal representation reflects the specific place value of each digit. When converting fractions to decimals, they carry out their calculations to specific place values, indicating a terminating or repeating pattern. In stating answers to problems involving signed numbers, students use integer rules and properties of operations to verify that the sign of their answer is correct. For instance, when finding an average temperature for temperatures whose sum is a negative number, students realize that the quotient must be a negative number since the divisor is positive and the dividend is negative. **MP.7 Look for and make use of structure.** Students formulate rules for operations with signed numbers by observing patterns. For instance, they notice that adding −7 to a number is the same as subtracting 7 from the number, and thus, they develop a rule for subtraction that relates to adding the inverse of the subtrahend. Students use the concept of absolute value and subtraction to represent the distance between two rational numbers on a number line. They use patterns related to the properties of operations to justify the rules for multiplying and dividing signed numbers. The order of operations provides the structure by which students evaluate and generate equivalent expressions.
Unit 1: Suggested Educational Resources	The student materials consist of the student pages for each lesson in Module 1. The copy ready materials are a collection of the module assessments, lesson exit tickets and fluency exercises from the teacher materials. **Websites to use:** **Khanacademy.org** **Flocabulary.com**		

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<u>Unit 2</u> **Rational Numbers (30 days)**	**Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.** **7.NS.1** Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.* b. Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. d. Apply properties of operations as strategies to add and subtract rational numbers. **7.NS.2** Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. a. Understand that	**Grade 7 Module 2: Rational Numbers** In Grade 6, students formed a conceptual understanding of integers through the use of the number line, absolute value, and opposites and extended their understanding to include the ordering and comparing of rational numbers. This module uses the Integer Game: a card game that creates a conceptual understanding of integer operations and serves as powerful mental model students can rely on during the module. Students build on their understanding of rational numbers to add, subtract, multiply, and divide signed numbers. Previous work in computing the sums, differences, products, and quotients of fractions serves as a significant foundation. The student materials consist of the student pages for each lesson in Module 2. The copy ready materials are a collection of the module assessments, lesson exit tickets and fluency exercises from the teacher materials. • In Topic A, students return to the number line to model the addition and subtraction of integers (7.NS.A.1). They use the number line and the Integer Game to demonstrate that an integer added to its opposite equals zero, representing the additive inverse	

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	multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero. Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers. 7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. a. Describe situations in which opposite quantities combine to make 0. For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged. b. Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. c. Understand subtraction of rational numbers as adding the	(7.NS.A.1a, 7.NS.A.1b). Their findings are formalized as students develop rules for adding and subtracting integers, and they recognize that subtracting a number is the same as adding its opposite (7.NS.A.1c). Real-life situations are represented by the sums and differences of signed numbers. Students extend integer rules to include the rational numbers and use properties of operations to perform rational number calculations without the use of a calculator (7.NS.A.1d). • Students develop the rules for multiplying and dividing signed numbers in Topic B. They use the properties of operations and their previous understanding of multiplication as repeated addition to represent the multiplication of a negative number as repeated subtraction (**7.NS.A.2a**). Students make analogies to the Integer Game to understand that the product of two negative numbers is a positive number. From earlier grades, they recognize division as the inverse process of multiplication. Thus, signed number rules for division are consistent with those for multiplication, provided a divisor is not zero (**7.NS.A.2b**). Students represent the division of two integers as a fraction, extending product and quotient rules to all rational numbers. They realize that any rational number in fractional form can be represented as a	

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	additive inverse, p – q = p + (–q). Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. d. Apply properties of operations as strategies to add and subtract rational numbers. 7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then –(p/q) = (–p)/q = p/(–q). Interpret quotients of rational numbers by describing real-world contexts. c. Apply properties of operations as strategies to multiply and divide rational numbers.ivisor) is a rational number. If *p* and *q* are integers, then *–(p/q) = (–p)/q = p/(–q).* Interpret quotients of rational numbers by describing real-	decimal that either terminates in 0s or repeats (**7.NS.A.2d**). Students recognize that the context of a situation often determines the most appropriate form of a rational number, and they use long division, place value, and equivalent fractions to fluently convert between these fraction and decimal forms. Topic B concludes with students multiplying and dividing rational numbers using the properties of operations (**7.NS.A.2c**). • In Topic C, students problem-solve with rational numbers and draw upon their work from Grade 6 with expressions and equations (**6.EE.A.2**, **6.EE.A.3**, **6.EE.A.4**, **6.EE.B.5**, **6.EE.B.6**, **6.EE.B.7**). They perform operations with rational numbers (**7.NS.A.3**), incorporating them into algebraic expressions and equations. They represent and evaluate expressions in multiple forms, demonstrating how quantities are related (**7.EE.A.2**). The Integer Game is revisited as students discover "if-then" statements, relating changes in player's hands (who have the same card-value totals) to changes in both sides of a number sentence. Students translate word problems into algebraic equations and become proficient at solving equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r, are	

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	world contexts. c. Apply properties of operations as strategies to multiply and divide rational numbers. d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats. **7.NS.3** Solve real-world and mathematical problems involving the four operations with rational numbers.18 **Use properties of operations to generate equivalent expressions.19** **7.EE.220** Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."* **Solve real-life and mathematical problems using numerical and algebraic expressions and equations.21** **7.EE.422** Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats. 7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers.18 Use properties of operations to generate equivalent	specific rational numbers (**7.EE.B.4a**). As they become fluent in generating algebraic solutions, students identify the operations, inverse operations, and order of steps, comparing these to an arithmetic solution. Use of algebra to represent contextual problems continues in Module 3.	

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	expressions.19 7.EE.220 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05." Solve real-life and mathematical problems using numerical and algebraic expressions and equations.21 7.EE.422 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form px + q = r and p(x + q) = r, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. Ford inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For*		

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Unit 2: Suggested Educational Resources	The student materials consist of the student pages for each lesson in Module 2. The copy ready materials are a collection of the module assessments, lesson exit tickets and fluency exercises from the teacher materials. **Websites to use:** **Khanacademy.org** **Flocabulary.com**		
Unit 3 **Expressions and Equations (35 days)** **In this module the equations include negative rational numbers.**	<u>Use properties of operations to generate equivalent expressions.</u> 7.EE.1 Apply properties of operations as strategies **7.EE.2** Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."* **Solve real-life and mathematical problems using numerical and algebraic expressions and equations.** **7.EE.323** Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to	In Topic C, students problem-solve with rational numbers and draw upon their work from Grade 6 with expressions and equations (**6.EE.A.2**, **6.EE.A.3**, **6.EE.A.4**, **6.EE.B.5**, **6.EE.B.6**, **6.EE.B.7**). They perform operations with rational numbers (**7.NS.A.3**), incorporating them into algebraic expressions and equations. They represent and evaluate expressions in multiple forms, demonstrating how quantities are related (**7.EE.A.2**). The Integer Game is revisited as students discover "if-then" statements, relating changes in player's hands (who have the same card-value totals) to changes in both sides of a number sentence. Students translate word problems into algebraic equations and become proficient at solving equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r, are specific rational numbers (**7.EE.B.4a**). As they become fluent in generating algebraic solutions,	**Unit 3:** **MP.2 Reason abstractly and quantitatively**. Students make sense of how quantities are related within a given context and formulate algebraic equations to represent this relationship. They use the properties of operations to manipulate the symbols that are used in place of numbers, in particular, pi. In doing so, students reflect upon each step in solving and recognize that these properties hold true since the variable is really just holding the place for a number. Students analyze solutions and connect back to ensure reasonableness within context. **MP.4 Model with mathematics**. Throughout the module, students use equations and inequalities as models to solve mathematical and real-world problems. In discovering the relationship between circumference and diameter in a circle, they will use real objects to analyze the relationship and draw conclusions. Students test conclusions with a variety of objects to see if the results hold true, possibly improving the model if it has not served its purpose. **MP.6 Attend to precision**. Students are precise in defining

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	calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.* **7.EE.4** Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?* b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and	students identify the operations, inverse operations, and order of steps, comparing these to an arithmetic solution. Use of algebra to represent contextual problems continues in Module 3. • In Topic C, students problem-solve with rational numbers and draw upon their work from Grade 6 with expressions and equations (**6.EE.A.2**, **6.EE.A.3**, **6.EE.A.4**, **6.EE.B.5**, **6.EE.B.6**, **6.EE.B.7**). They perform operations with rational numbers (**7.NS.A.3**), incorporating them into algebraic expressions and equations. They represent and evaluate expressions in multiple forms, demonstrating how quantities are related (**7.EE.A.2**). The Integer Game is revisited as students discover "if-then" statements, relating changes in player's hands (who have the same card-value totals) to changes in both sides of a number sentence. Students translate word problems into algebraic equations and become proficient at solving equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r, are specific rational numbers (**7.EE.B.4a**). As they become fluent in generating algebraic solutions, students identify the operations, inverse operations, and order of steps, comparing these to an arithmetic solution. Use of algebra to represent contextual problems	variables. They understand that a variable represents one number. They use appropriate vocabulary and terminology when communicating about expressions, equations, and inequalities. They use the definition of equation from Grade 6 to understand how to use the equal sign consistently and appropriately. Circles and related notions about circles are precisely defined in this module. **MP.7 Look for and make use of structure.** Students recognize the repeated use of the distributive property as they write equivalent expressions. Students recognize how equations leading to the form $pppp+qq=rr$ and $(xx+qq)=rr$ are useful in solving a variety of problems. They see patterns in the way that these equations are solved. Students apply this structure as they understand the similarities and differences in how an inequality of the type $pppp+qq>rr$ or $pppp+qq<rr$ is solved. **MP.8 Look for and express regularity in repeated reasoning.** Students use area models to write products as sums and sums as products and recognize how this model is a way to organize results from repeated use of the distributive property. As students work to solve problems, they maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of solutions as they are represented in contexts that allow for students to know that they found the intended value for a given variable. As they solve problems involving pi, they notice how a problem may be reduced by using a given estimate for pi to make calculations more efficient. **Unit 4:** **MP.1 Make sense of problems and persevere in solving them.** Students make sense of percent problems by modeling the proportional relationship using an equation, a table, a graph, a double number line diagram, mental math, and factors of 100. When solving a multi-step percent word problem, students use estimation and number sense to determine if their steps and logic lead to a reasonable answer. Students know they can

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	interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.* **Solve real-life and mathematical problems involving angle measure, area, surface area, and volume. 24** **7.G.4** Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle. **7.G.5** Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure. **7.G.6** Solve real-world and mathematical problems involving area, volume and surface area of two- and three-	continues in Module 3. • In Topic C, students problem-solve with rational numbers and draw upon their work from Grade 6 with expressions and equations (**6.EE.A.2**, **6.EE.A.3**, **6.EE.A.4**, **6.EE.B.5**, **6.EE.B.6**, **6.EE.B.7**). They perform operations with rational numbers (**7.NS.A.3**), incorporating them into algebraic expressions and equations. They represent and evaluate expressions in multiple forms, demonstrating how quantities are related (**7.EE.A.2**). The Integer Game is revisited as students discover "if-then" statements, relating changes in player's hands (who have the same card-value totals) to changes in both sides of a number sentence. Students translate word problems into algebraic equations and become proficient at solving equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r, are specific rational numbers (**7.EE.B.4a**). As they become fluent in generating algebraic solutions, students identify the operations, inverse operations, and order of steps, comparing these to an arithmetic solution. Use of algebra to represent contextual problems continues in Module 3. In Topic C, Students continue work with geometry as they use equations and expressions to study area, perimeter,	always find 1% of a quantity by dividing it by 100 or multiplying it by 1100, and they also know that finding 1% first allows them to then find other percents easily. For instance, if students are trying to find the amount of money after 4 years in a savings account with an annual interest rate of 12% on an account balance of $300, they use the fact that 1% of 300 equals 300100, or $3; thus, 12% of 300 equals 12 of $3, or $1.50. $1.50 multiplied by 4 is $6 interest, and adding $6 to $300 makes the total balance, including interest, equal to $306. **MP.2 Reason abstractly and quantitatively.** Students use proportional reasoning to recognize that when they find a certain percent of a given quantity, the answer must be greater than the given quantity if they found more than 100% of it and less than the given quantity if they found less than 100% of it. Double number line models are used to visually represent proportional reasoning related to percent in problems such as the following: If a father has 70% more money in his savings account than his 25-year-old daughter has in her savings account, and the daughter has $4,500, how much is in the father's account? Students represent this information with a visual model by equating 4,500 to 100% and the father's unknown savings amount to 170% of 4,500. Students represent the amount of money in the father's savings account by writing the expression 170100×4,500, or 1.7(4,500). When working with scale drawings, given an original two-dimensional picture and a scale factor as a percent, students generate a scale drawing so that each corresponding measurement increases or decreases by a certain percentage of measurements of the original figure. Students work backward to create a new scale factor and scale drawing when given a scale factor represented as a percent greater or less than 100%. For instance, given a scale drawing with a scale factor of 25%, students create a new scale drawing with a scale factor of 10%. They relate working backward in their visual model to the following steps: (1) multiplying all lengths in the original scale drawing by 10.25 (or

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		surface area, and volume. This final topic begins by modeling a circle with a bicycle tire and comparing its perimeter (one rotation of the tire) to the length across (measured with a string) to allow students to discover the most famous ratio of all, pi. Activities in comparing circumference to diameter are staged precisely for students to recognize that this symbol has a distinct value and can be approximated by 22/7 or 3.14 to give students an intuitive sense of the relationship that exists. In addition to representing this value with the pi symbol, the fraction and decimal approximations allow for students to continue to practice their work with rational number operations. All problems are crafted in such a way to allow students to practice skills in reducing within a problem, such as using 22/7 for finding circumference with a given diameter length of 14 cm, and recognize what value would be best to approximate a solution. This understanding allows students to accurately assess work for reasonableness of answers. After discovering and understanding the value of this special ratio, students will continue to use pi as they solve problems of area and circumference (7.G.B.4). In this topic, students derive the formula for area of a circle by dividing a circle of radius *r* into pieces of pi and rearranging the pieces so that they are lined up, alternating direction, and form a shape	dividing by 25%) to get back to their original lengths and then (2) multiplying each original length by 10% to get the new scale drawing. **MP.5 Use appropriate tools strategically.** Students solve word problems involving percents using a variety of tools, including equations and double number line models. They choose their model strategically. For instance, given that 75% of a class of learners is represented by 21 students, they recognize that since 75 is 34 of 100, and 75 and 21 are both divisible by 3, a double number line diagram can be used to establish intervals of 25's and 7's to show that 100% would correspond to 21+7, which equals 28. For percent problems that do not involve benchmark fractions, decimals, or percents, students use math sense and estimation to assess the reasonableness of their answers and computational work. For instance, if a problem indicates that a bicycle is marked up 18% and is sold at a retail price of $599, students are able to estimate by using rounded values such as 120% and $600 to determine that the solution that represents the wholesale price of the bicycle must be in the realm of 600÷1.2, or 6,000÷12, to arrive at an estimate of $500. **MP.6 Attend to precision.** Students pay close attention to the context of the situation when working with percent problems involving a percent markup, markdown, increase, or decrease. They construct models based on the language of a word problem. For instance, the total cost of an $88 item marked down by 15% is represented by 0.85(88); however, the total cost of the same item marked up by 15% is represented by 1.15(88). Students attend to precision when writing the answer to a percent problem. If they are finding a percent, they use the % symbol in the answer or write the answer as a fraction with 100 as the denominator (or in an equivalent form). Double number line diagrams display correct segment lengths, and if a line in the diagram represents percents, it is either labeled as such or the percent sign is shown after each number. When stating the area of a scale drawing or actual drawing, students include the square units along with the

Overview	Standards for Mathematical Content	Unit Focus	Standards for Mathematical Practice
		that resembles a rectangle. This "rectangle" has a length that is 1/2 the circumference and a width of r. Students determine that the area of this rectangle (reconfigured from a circle of the same area) is the product of its length and its width: $1/2(C)(r) = 1/2\ 2(\text{pi})(r)(r) = \text{pi}(r)^2$ (**7.G.B.4**). The precise definitions for diameter, circumference, pi, and circular region or disk will be developed during this topic with significant time being devoted to student understanding of each term. Students build upon their work in Grade 6 with surface area and nets to understand that surface area is simply the sum of the area of the lateral faces and the base(s) (6.G.A.4). In Grade 7, they continue to solve real-life and mathematical problems involving area of two-dimensional shapes and surface area and volume of prisms, e.g., rectangular, triangular, focusing on problems that involve fractional values for length (**7.G.B.6**). Additional work (examples) with surface area will occur in Module 6 after a formal definition of rectangular pyramid is established. •	numerical part of the answer. **MP.7 Look for and make use of structure**. Students understand percent to be a rate per 100 and express pp percent as $pp100$. They know that, for instance, 5% means 5 for every 100, 1% means 1 for every 100, and 225% means 225 for every 100. They use their number sense to find benchmark percents. Since 100% is one whole, then 25% is one-fourth, 50% is one-half, and 75% is three-fourths. So, to find 75% of 24, they find 14 of 24, which is 6, and multiply it by 3 to arrive at 18. They use factors of 100 and mental math to solve problems involving other benchmark percents as well. Students know that 1% of a quantity represents 1100 of it and use place value and the structure of the base-ten number system to find 1% or 1100 of a quantity. They use finding 1% as a method to solve percent problems. For instance, to find 14% of 245, students first find 1% of 245 by dividing 245 by 100, which equals 2.45. Since 1% of 245 equals 2.45, 14% of 245 would equal $2.45\times14=34.3$. Students observe the steps involved in finding a discount price or price including sales tax and use the properties of operations to efficiently find the answer. To find the discounted price of a $73 item that is on sale for 15% off, students realize that the distributive property allows them to arrive at an answer in one step, by multiplying $73 by 0.85, since $73(100\%)-73(15\%)=73(1)-73(0.15)=73(0.85)$.
Unit 3: Suggested Educational Resources	The student materials consist of the student pages for each lesson in Module 3. The copy ready materials are a collection of the		

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	module assessments, lesson exit tickets and fluency exercises from the teacher materials. **Websites to use:** **Khanacademy.org** **Flocabulary.com**		
<u>**Unit 4**</u> **Percent and Proportional Relationships** **(25 days)** **The emphasis in this module is on percent.**	Analyze proportional relationships and use them to solve real-world and mathematical problems. <u>**7.RP.1**</u> Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2 / 1/4 miles per hour, equivalently 2 miles per hour. <u>**7.RP.2**</u> Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a	**Grade 7 Module 4: Percent and Proportional Relationships** • In Module 4, students deepen their understanding of ratios and proportional relationships from Module 1 by solving a variety of percent problems. They convert between fractions, decimals, and percents to further develop a conceptual understanding of percent and use algebraic expressions and equations to solve multi-step percent problems. An initial focus on relating 100% to "the whole" serves as a foundation for students. Students begin the module by solving problems without using a calculator to develop an understanding of the reasoning underlying the calculations. Material in early lessons is designed to	

Overview	Standards for Mathematical Content	Unit Focus	Standards for Mathematical Practice
	proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn. d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate. **7.RP.3** Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent	reinforce students' understanding by having them use mental math and basic computational skills. To develop a conceptual understanding, students use visual models and equations, building on their earlier work with these. As the lessons and topics progress and students solve multi-step percent problems algebraically with numbers that are not as compatible, teachers may let students use calculators so that their computational work does not become a distraction. The student materials consist of the student pages for each lesson in Module 4. The copy ready materials are a collection of the module assessments, lesson exit tickets and fluency exercises from the teacher materials. • Topic A (lessons 1-6) builds on students' conceptual understanding of percent from Grade 6 (**6.RP.3c**), and relates 100% to "the whole." Students represent percents as decimals and fractions and extend their understanding from Grade 6 to include percents greater than 100%, such as 225%, and percents less than 1%, such as 1/2% or 0.5%. They understand that, for instance, 225% means 225/100 , or equivalently, 2.25/1 =	

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	increase and decrease, percent error. Solve real-life and mathematical problems using numerical and algebraic expressions and equations. **<u>7.EE.3</u>** Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example: If a woman making \$25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or \$2.50, for a new salary of \$27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation. Draw, construct, and describe geometrical figures and	2.25 (**7.RP.A.1**). Students use complex fractions to represent non-whole number percents. Module 3's focus on algebra prepares students to move from the visual models used for percents in Grade 6 to algebraic equations in Grade 7. They write equations to solve multi-step percent problems and relate their conceptual understanding to the representation: *Quantity* = *Percent* × *Whole* (**7.RP.A.2c**). Students solve percent increase and decrease problems with and without equations (**7.RP.A.3**). For instance, given a multi-step word problem where there is an increase of 20% and "the whole" equals \$200, students recognize that \$200 can be multiplied by 120% or 1.2 to get an answer of \$240. They use visual models, such as a double number line diagram, to justify their answers. In this case, 100% aligns to \$200 in the diagram and intervals of fifths are used (since 20% = 1/5) to partition both number line segments to create a scale indicating that 120% aligns to \$240. Topic A concludes with students representing 1% of a quantity using a ratio, and then using that ratio to find the amounts of other percents. While representing 1% of a quantity and using it to find the amount of other percents is a strategy that will always work when solving a problem, students recognize that when the percent	

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	describe the relationships between them. **7.G.1** Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.	is a factor of 100, they can use mental math and proportional reasoning to find the amount of other percents. In Topic B, students create algebraic representations and apply their understanding of percent from Topic A to interpret and solve multi-step word problems related to markups or markdowns, simple interest, sales tax, commissions, fees, and percent error (**7.RP.A.3**, **7.EE.B.3**). They apply their understanding of proportional relationships from Module 1, creating an equation, graph, or table to model a tax or commission rate that is represented as a percent (**7.RP.A.1**, **7.RP.A.2**). Students solve problems related to changing percents and use their understanding of percent and proportional relationships to solve the following: *A soccer league has 300 players, 60% of whom are boys. If some of the boys switch to baseball, leaving only 52% of the soccer players as boys, how many players remain in the soccer league?* Students determine that, initially, 100% = 60% = 40% of the players are girls and 40% of 300 equals 120. Then, after some boys switched to baseball, 100% - 52% of the soccer players are girls, so $0.48p = 120$, or $p = 120/0.48$. Therefore, there are now 250 players in the soccer league. In Topic B,(lessons 6-11) students also apply their understanding of absolute value from Module 2 (**7.NS.A.1b**) when	

Overview	Standards for Mathematical Content	Unit Focus	Standards for Mathematical Practice
		solving percent error problems. To determine the percent error for an estimated concert attendance of 5,000 people, when actually 6,372 people attended, students calculate the percent error as: (|5000 - 6372|)/|6372|, which is about 21.5%. • Students revisit scale drawings in Topic C (lessons 12-15) to solve problems in which the scale factor is represented by a percent (**7.RP.A.2b**, **7.G.A.1**). They understand from their work in Module 1, for example, that if they have two drawings where if Drawing 2 is a scale model of Drawing 1 under a scale factor of 80%, then Drawing 1 is also a scale model of Drawing 2, and that scale factor is determined using inverse operations. Since 80% = 4/5, the scale factor is found by taking the complex fraction (1)/(4/5), or 5/4, and multiplying it by 100%, resulting in a scale factor of 125%. As in Module 1, students construct scale drawings, finding scale lengths and areas given the actual quantities and the scale factor (and vice-versa); however, in this module the scale factor is represented as a percent. Students are encouraged to develop multiple methods for making scale drawings. Students may find the multiplicative relationship between figures; they may also find a multiplicative relationship among lengths within the same figure. • The problem-solving material in Topic D (lessons 16-18) provides students with	

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		further applications of percent and exposure to problems involving population, mixtures, and counting, in preparation for later topics in middle school and high school mathematics and science. Students apply their understanding of percent (**7.RP.A.2c**, **7.RP.A.3**, **7.EE.B.3**) to solve word problems in which they determine, for instance, when given two different sets of 3-letter passwords and the percent of 3-letter passwords that meet a certain criteria, which set is the correct set. Or, given a 12-gallon mixture that is 40% pure juice, students determine how many gallons of pure juice must be added to create a 5-gallon mixture that is 20% pure juice by writing and solving the equation $0.2(5) + j = 0.4(12)$, where j is the amount of pure juice added to the original mixture.	
Unit 4: Suggested Educational Resources	The student materials consist of the student pages for each lesson in Module 4. The copy ready materials are a collection of the module assessments, lesson exit tickets and fluency exercises from the teacher materials. **Websites to use:** **Khanacademy.org** **Flocabulary.com**		

Unit 5 **Statistics and Probability** (25 days)	**Use random sampling to draw inferences about a population.** **7.SP.1** Understand that statistics can be used to gain information about a	**Grade 7 Module 5: Statistics and Probability** In this module, students begin their study of probability, learning how to interpret probabilities and how to compute probabilities in simple settings. They also learn how to estimate probabilities	**Unit 5:** **MP.2 Reason abstractly and quantitatively.** Students reason quantitatively by posing statistical questions about variables and the relationship between variables. Students

	population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences. **7.SP.2** Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.* **Draw informal comparative inferences about two populations.** **7.SP.3** Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the	empirically. Probability provides a foundation for the inferential reasoning developed in the second half of this module. Additionally, students build on their knowledge of data distributions that they studied in Grade 6, compare data distributions of two or more populations, and are introduced to the idea of drawing informal inferences based on data from random samples. The student materials consist of the student pages for each lesson in Module 5. The copy ready materials are a collection of the module assessments, lesson exit tickets and fluency exercises from the teacher materials. In Topics A ,(lessons 1-7) students learn to interpret the probability of an event as the proportion of the time that the event will occur when a chance experiment is repeated many times (**7.SP.C.5**). They learn to compute or estimate probabilities using a variety of methods, including collecting data, using tree diagrams, and using simulations. In Topic B, (lessons 8-12) students move to comparing probabilities from simulations to computed probabilities that are based on theoretical models (**7.SP.C.6**, **7.SP.C.7**). They calculate probabilities of compound events using lists, tables, tree diagrams, and simulations (**7.SP.C.8**). They learn to use probabilities to make decisions and to determine whether or not a given probability model is plausible (**7.SP.C.7**). The Mid-Module Assessment follows Topic B. In Topics C , (lessons 13-20) students focus on using random sampling to draw informal inferences about a population (**7.SP.A.1**, **7.SP.A.2**). In Topic C, they investigate sampling from a population (**7.SP.A.2**). They learn to estimate a population mean using numerical data from a random sample (**7.SP.A.2**). They also learn how to estimate a population proportion using categorical data from a random	reason abstractly about chance experiments by analyzing possible outcomes and designing simulations to estimate probabilities. **MP.3 Construct viable arguments and critique the reasoning of others.** Students construct viable arguments by using sample data to explore conjectures about a population. Students critique the reasoning of other students as part of poster or similar presentations. **MP.4 Model with mathematics.** Students use probability models to describe outcomes of chance experiments. They evaluate probability models by calculating the theoretical probabilities of chance events and by comparing these probabilities to observed relative frequencies. **MP.5 Use appropriate tools strategically.** Students use simulation to approximate probabilities. Students use appropriate technology to calculate measures of center and variability. Students use graphical displays to visually represent distributions. **MP.6 Attend to precision.** Students interpret and communicate conclusions in context based on graphical and numerical

	centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.* **7.SP.4** Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.* **Investigate chance processes and develop, use, and evaluate probability models.** **7.SP.5** Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a	sample. In Topic D, (lessons 21-23)students learn to compare two populations with similar variability. They learn to consider sampling variability when deciding if there is evidence that the means or the proportions of two populations are actually different (**7.SP.B.3**, **7.SP.B.4**). The End-of-Module Assessment follows Topic D.	

	probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event. **7.SP.6** Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.* **7.SP.7** Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a*		

	girl will be selected. b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?* **7.SP.8** Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event. c. Design and use a		

	simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*		
Unit 5: Suggested Educational Resources	The student materials consist of the student pages for each lesson in Module 5. The copy ready materials are a collection of the module assessments, lesson exit tickets and fluency exercises from the teacher materials. **Websites to use:** **Khanacademy.org** **Flocabulary.com**		
Unit 6 **Geometry** (35 days)	**Draw, construct, and describe geometrical figures and describe the relationships between them.28** **7.G.2** Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. **7.G.3** Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids. **Solve real-life and mathematical**	In Module 6, students delve further into several geometry topics they have been developing over the years. Grade 7 presents some of these topics (e.g., angles, area, surface area, and volume) in the most challenging form students have experienced yet. Module 6 assumes students understand the basics; the goal is to build fluency in these difficult problems. The remaining topics (i.e., working on constructing triangles and taking slices, or cross sections, of three-dimensional figures) are new to students. In Topic A, (lessons 1-4) students solve for unknown angles. The supporting work for unknown angles began in Grade 4 Module 4 (**4.MD.C.5**, **4.MD.C.6**, **4.MD.C.7**), where all of the key terms in this topic were first defined, including the following: adjacent, vertical, complementary, and supplementary angles; angles on a line; and angles at a point. In Grade 4, students used those definitions as a basis to solve for unknown angles by using a combination of reasoning (through simple number sentences and equations) and measurement (using a protractor). For example,	**Unit 6:** **MP.1 Make sense of problems and persevere in solving them**. This mathematical practice is particularly applicable for this module, as students tackle multi-step problems that require them to tie together knowledge about their current and former topics of study (i.e., a real-life composite area question that also requires proportions and unit conversion). In many cases, students have to make sense of new and different contexts and engage in significant struggle to solve problems. **MP.3 Construct viable arguments and critique the reasoning of others**. In Topic B, students examine the conditions that determine a unique triangle, more than one triangle, or no triangle. They have the opportunity to defend and critique the

	problems involving angle measure, area, surface area, and volume.29 **7.G.5** Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure. **7.G.6** Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	students learned to solve for an unknown angle in a pair of supplementary angles where one angle measurement is known. In Grade 7 Module 3, students studied how expressions and equations are an efficient way to solve problems. Two lessons were dedicated to applying the properties of equality to isolate the variable in the context of unknown angle problems. The diagrams in those lessons were drawn to scale to help students more easily make the connection between the variable and what it actually represents. Now in Module 6, the most challenging examples of unknown angle problems (both diagram-based and verbal) require students to use a synthesis of angle relationships and algebra. The problems are multi-step, requiring students to identify several layers of angle relationships and to fit them with an appropriate equation to solve. Unknown angle problems show students how to look for, and make use of, structure (MP.7). In this case, they use angle relationships to find the measurement of an angle. Next, in Topic B, (lessons 5-15) students work extensively with a ruler, compass, and protractor to construct geometric shapes, mainly triangles (**7.G.A.2**). The use of a compass is new (e.g., how to hold it and how to create equal segment lengths). Students use the tools to build triangles with given conditions such as side length and the measurement of the included angle (MP.5). Students also explore how changes in arrangement and measurement affect a triangle, culminating in a list of conditions that determine a unique triangle. Students understand two new concepts about unique triangles. They learn that under a condition that determines a unique triangle: (1) a triangle can be drawn, and (2) any two triangles drawn under the condition are identical. It is important to note that there is no mention of congruence in the CCSS until Grade 8, after a study of rigid motions. Rather, the focus of Topic B is developing students' intuitive understanding of the structure of a triangle. This includes students noticing the conditions that determine a unique triangle, more than one triangle, or no triangle (**7.G.A.2**). Understanding what makes triangles unique requires understanding what makes them identical.	reasoning of their own arguments as well as the arguments of others. In Topic C, students predict what a given slice through a three-dimensional figure yields (i.e., how to slice a three-dimensional figure for a given cross section) and must provide a basis for their predictions. **MP.5 Use appropriate tools strategically**. In Topic B, students learn how to strategically use a protractor, ruler, and compass to build triangles according to provided conditions. An example of this is when students are asked to build a triangle provided three side lengths. Proper use of the tools helps them understand the conditions by which three side lengths determine one triangle or no triangle. Students have opportunities to reflect on the appropriateness of a tool for a particular task. **MP.7 Look for and make use of structure**. Students must examine combinations of angle facts within a given diagram in Topic A to create an equation that correctly models the angle relationships. If the unknown angle problem is a verbal problem, such as an example that asks for the measurements of three angles on a line where the values of the measurements are consecutive numbers, students have to create an equation without a visual aid and rely on the inherent structure of the angle fact. In Topics D and E, students find area, surface area, and volume of composite figures based on the structure of two- and three-dimensional figures.

		Topic C, (lessons 16-19) introduces the idea of a slice, or cross section, of a three-dimensional figure. Students explore the two-dimensional figures that result from taking slices of right rectangular prisms and right rectangular pyramids parallel to the base and parallel to a lateral face; they also explore two-dimensional figures that result from taking skewed slices that are not parallel to either the base or a lateral face (**7.G.A.3**). The goal of the first three lessons is to get students to consider three-dimensional figures from a new perspective. One way students do this is by experimenting with an interactive website that requires students to choose how to position a three-dimensional figure so that a slice yields a particular result (e.g., how a cube should be sliced to get a pentagonal cross section). Similar to Topic A, the subjects of area, surface area, and volume in Topics D and E (lessons 20-27) are not new to students but provide opportunities for students to expand their knowledge by working with challenging applications. In Grade 6, students verified that the volume of a right rectangular prism is the same whether it is found by packing it with unit cubes or by multiplying the edge lengths of the prism (**6.G.A.2**). In Grade 7, the volume formula $V = Bh$, where B represents the area of the base, is tested on a set of three-dimensional figures that extends beyond right rectangular prisms to right prisms in general. In Grade 6, students practiced composing and decomposing two-dimensional shapes into shapes they could work with to determine area (**6.G.A.1**). Now, they learn to apply this skill to volume as well. The most challenging problems in these topics are not pure area or pure volume questions but problems that incorporate a broader mathematical knowledge such as rates, ratios, and unit conversion. It is this use of multiple skills and contexts that distinguishes real-world problems from purely mathematical ones (**7.G.B.6**).	

Unit 6: Suggested Educational Resources	The student materials consist of the student pages for each lesson in Module 6 The copy ready materials are a collection of the module assessments, lesson exit tickets and fluency exercises from the teacher materials. **Websites to use:** **Khanacademy.org** **Flocabulary.com**

Unit 1 Vocabulary

Terminology

New or Recently Introduced Terms

- **Constant of Proportionality** (If a proportional relationship is described by the set of ordered pairs that satisfies the equation $yy=kkkk$, where kk is a positive constant, then kk is called the *constant of proportionality*. For example, if the ratio of yy to xx is 2 to 3, then the constant of proportionality is 23 , and $yy=23xx$.)
- **Miles per Hour** (One *mile per hour* is a proportional relationship between dd miles and tt hours given by the equation $dd=1\cdot tt$ (both dd and tt are positive real numbers). Similarly, for any positive real number vv, *vv miles per hour* is a proportional relationship between dd miles and tt hours given by $dd=vv\cdot tt$. The unit for the rate, mile per hour (or mile/hour) is often abbreviated as mph.)
- **One-To-One Correspondence Between Two Figures in the Plane (description)** (For two figures in the plane, SS and SS', a *one-to-one correspondence between the figures* is a pairing between the points in SS and the points in SS' so that each point PP of SS is paired with one and only one point PP' in SS', and likewise, each point QQ' in SS' is paired with one and only one point QQ in SS.)

Proportional Relationship (description) (A *proportional relationship* is a correspondence between two types of quantities such that the measures of quantities of the first type are proportional to the measures of quantities of the second type.
Note that proportional relationships and ratio relationships describe the same set of ordered pairs but in two different ways. Ratio relationships are used in the context of working with equivalent ratios, while proportional relationships are used in the context of rates.)

- **Proportional To (description)** (Measures of one type of quantity are *proportional to* measures of a second type of quantity if there is a number kk so that for every measure xx of a quantity of the first type, the corresponding measure yy of a quantity of the second type is given by $kkkk$; that is, $yy=kkkk$. The number kk is called the constant of proportionality.)
- **Scale Drawing and Scale Factor (description)** (For two figures in the plane, SS and SS', SS' is said to be a *scale drawing* of SS with *scale factor* rr if there exists a one-to-one correspondence between SS and SS' so that, under the pairing of this one-to-one correspondence, the distance $|PPPP|$ between any two points PP and QQ of SS is related to the distance $|PP'QQ'|$ between corresponding points PP' and QQ' of SS' by $|PP'QQ'|=rr|PPPP|$.)

Familiar Terms and Symbols

- Equivalent Ratio
- Rate
- Ratio
- Ratio Table

- Unit Rate

Unit 2 Vocabulary

Terminology

New or Recently Introduced Terms

- **Additive Identity** (The additive identity is the number 0.)
- **Additive Inverse** (An *additive inverse of a number* is a number such that the sum of the two numbers is 0. The additive inverse of a number aa is the opposite of aa (i.e., $-aa$) because if we add both numbers using the vector approach above, we get $aa+(-aa)=0$.)
- **Formula for the Distance Between Two Numbers** (If pp and qq are numbers on a number line, then the *distance between* pp *and* qq is $|pp-qq|$.)
- **Multiplicative Identity** (The *multiplicative identity* is the number 1.)
- **Repeating Decimal Expansion** (A decimal expansion is *repeating* if, after some digit to the right of the decimal point, there is a finite string of consecutive digits called a block after which the decimal expansion consists entirely of consecutive copies of that block repeated forever.

A common notation for a repeating decimal expansion is to write out the decimal expansion until the end of the first block and then writing a line over the block. For example, 3.125 is a compact way to write the repeating decimal expansion 3.125252525....)

- **Terminating Decimal Expansion** (A *terminating decimal expansion* is a repeating decimal expansion with period 1 and repeating digit 0. For example, the terminating decimal expansion of 14 is 0.25000..., or 0.250. A number with a terminating decimal expansion can be written as a finite decimal. For example, the terminating decimal expansion 0.25000... represents the same number as the finite decimal 0.25.)

Familiar Terms and Symbols

- Absolute Value
- Associative Property (of Multiplication and Addition)
- Commutative Property (of Multiplication and Addition)
- Credit
- Debit
- Deposit
- Distributive Property (of Multiplication Over Addition)
- Equation
- Expression
- Integer

- Inverse
- Multiplicative Inverse
- Negatives
- Opposites
- Overdraft
- Positives
- Rational Numbers
- Withdraw

Unit 3 Vocabulary

Terminology

New or Recently Introduced Terms

- **An Expression in Expanded Form (description)** (An expression that is written as sums (and/or differences) of products whose factors are numbers, variables, or variables raised to whole number powers is said to be in *expanded form*. A single number, variable, or a single product of numbers and/or variables is also considered to be in expanded form.)
- **An Expression in Factored Form (description)** (An expression that is a product of two or more expressions is said to be in *factored form*.)
- **An Expression in Standard Form (description)** (An expression that is in expanded form where all like terms have been collected is said to be in *standard form*.)
- **Circle** (Given a point OO in the plane and a number $rr>0$, the *circle with center OO and radius rr* is the set of all points in the plane whose distance from the point OO is equal to rr.)
- **Circular Region or Disk** (Given a point CC in the plane and a number $rr>0$, the *circular region (or disk) with center CC and radius rr* is the set of all points in the plane whose distance from the point CC is less than or equal to rr.)

Circumference (The *circumference of a circle* is the distance around the circle.)3
- **Coefficient of a Term** (The *coefficient of a term* is the number found by multiplying all of the number factors in a term together.)
- **Diameter of a Circle** (The *diameter of a circle* is the length of any segment that passes through the center of a circle whose endpoints lie on the circle. If rr is the radius of a circle, then the diameter is $2rr$.)
- **Interior of a Circle** (The *interior of a circle with center CC and radius rr* is the set of all points in the plane whose distance from the point CC is less than rr.)
- **Pi** (The number *pi*, denoted $\pi\pi$, is the value of the ratio given by the circumference to the diameter, that is, $\pi\pi$ = (circumference)/(diameter).)
- **Term (description)** (Each summand of an expression in expanded form is called a *term*.)

Familiar Terms and Symbols

- Adjacent Angles
- Cube

- Distribute
- Equation
- Equivalent Expressions
- Expression (middle school description)
- Factor
- Figure
- Identity
- Inequality
- Length of a Segment
- Linear Expression
- Measure of an Angle
- Number Sentence
- Numerical Expression (middle school description)
- Properties of Operations (distributive, commutative, associative)
- Right Rectangular Prism
- Segment
- Square
- Surface of a Prism
- Term
- Triangle

Unit 4 Vocabulary

Terminology

New or Recently Introduced Terms

- **Absolute Error** (Given the exact value xx of a quantity and an approximate value aa of it, the absolute error is $|aa-xx|$.)
- **Percent Error** (The *percent error* is the percent the absolute error is of the exact value, i.e., $|aa-xx||xx|\cdot 100\%$, where xx is the exact value of the quantity and aa is an approximate value of the quantity.)

Familiar Terms and Symbols

- Area
- Circumference
- Coefficient of the Term
- Complex Fraction
- Constant of Proportionality
- Discount Price
- Equation
- Equivalent Ratios
- Expression

- Fee
- Fraction
- Greatest Common Factor
- Length of a Segment
- One-to-One Correspondence
- Original Price
- Percent
- Perimeter
- Pi
- Proportional Relationship
- Proportional To
- Rate
- Ratio
- Rational Number
- Sales Price
- Scale Drawing
- Scale Factor
- Unit Rate

Unit 5 Vocabulary

Terminology New or Recently Introduced Terms

- **Chance Experiment (description)** (A *chance experiment* consists of observing a single outcome of a chance process.) ▪ **Chance Process (description)** (A *chance process* is any process that is repeatable and results in one of two or more well-defined outcomes each time it is repeated. The act of performing the process and producing a result is called a *trial*. In a chance process, trials are independent from each other in that the result of one trial does not influence the result of any other trial.

In the context of probability, observing a single outcome of a chance process is sometimes called a chance experiment.)

- **Event** (An *event* is a subset of outcomes of the sample space.

A *simple event* is an event with a single outcome. A *compound event* is an event that is the union of two or more simple events.)

- **Frequency of an Event** (The *frequency of an event* is the number of trials for which the event occurred in performing a fixed number of trials of a chance process.)
- **Long-Run Relative Frequency (description)** (The *long-run relative frequency* is the number that the relative frequencies get closer and closer to as more and more trials are performed.

The long-run relative frequency for a large number of trials provides an estimate of the probability of that event occurring that can be used when the probability of that event is hard to calculate from the probability model.)

- **Population (description)** (A *population* is any entire collection of people, animals, plants, or things that someone is interested in learning about. Each person or object in the population is called a *member*.) ▪ **Probability (description)** (The *probability of an event* is a number between 0 and 1 that measures the chance that the event will occur.) ▪ **Probability Model (description)** (A *probability model* is a mathematical representation of a chance process defined by its sample space, events within the sample space, and the assignment of a probability for each and every event.) ▪ **Probability Simulation (illustration)** (A *probability simulation* is the use of a random number generator (e.g., spinners, coin toss, computers) to generate outcomes that are consistent with a given probability model.

For example, to estimate the probability that a family with 4 children will include 3 or more boys, the children in a family might be represented by a sequence of four random digits with even digits representing a girl and odd digits representing a boy. This would generate "families" using a model that is consistent with a family having 4 children and each child being equally likely to be male or female. A large number of simulated "families" could be generated using technology, and then the relative frequency of those with 3 or more boys provides an estimate of the probability that a family with 4 children will include 3 or more boys.)

▪ **Random Sample** (A *random sample of size* nn is a sample that is selected using a process that ensures that every different possible sample of size nn had the same chance of being selected as the sample.

This selection process implies that every individual member of the population has the same chance of being included in the sample.)

▪ **Relative Frequency of an Event** (The *relative frequency of an event* is the value given by the frequency of the event divided by the total number of trials.)

▪ **Sample** (A *sample* is any subset of a population.) ▪ **Sample Space** (The *sample space* of a chance process is the set of all possible outcomes.

For example, the sample space for the experiment of rolling a die is the set $\{1,2,3,4,5,6\}$.)

▪ **Sample Statistic (description)** (A *sample statistic* or *statistic* is a number (e.g., mean, standard deviation) that is calculated from a sample.

Statistics are used to estimate, predict, or make decisions about population parameters.)

▪ **Statistical Inference** (*Statistical inference* is the process of drawing conclusions about population parameters using sample statistics.

In Grade 7, this can be described as "the process of drawing conclusions about populations using information from a sample of the population." The words in the definition above are described in this section.)

▪ **Uniform Probability Model** (A *uniform probability model* is a probability model that assigns the same probability for all simple events of the sample space.)

Familiar Terms and Symbols

▪ Mean Absolute Deviation (MAD) ▪ Measures of Center ▪ Measures of Variability ▪ Shape

Unit 6 Vocabulary

Terminology New or Recently Introduced Terms

- ▪ **Right Rectangular Pyramid** (Given a rectangular region B in a plane E and a point V not in E, the rectangular pyramid with base B and vertex V is the union of all segments VP for any point P in B.

  It can be shown that the planar region defined by a side of the base B and the vertex V is a triangular region called a lateral face. If the vertex lies on the line perpendicular to the base at its center (i.e., the intersection of the rectangle's diagonals), the pyramid is called a right rectangular pyramid.)
- **Surface of a Pyramid** (The *surface of a pyramid* is the union of its base region and its lateral faces.)
- **Three Sides Condition** (Two triangles satisfy the *three sides condition* if there is a triangle correspondence between the two triangles such that each pair of corresponding sides are equal in length.)
- **Triangle Correspondence** (A *triangle correspondence* between two triangles is a pairing of each vertex of one triangle with one and only one vertex of the other triangle.

  A triangle correspondence also induces a correspondence between the angles of the triangles and the sides of the triangles.)
- **Triangles with Identical Measures** (Two triangles are said to have *identical measures* if there is a triangle correspondence such that all pairs of corresponding sides are equal in length and all pairs of corresponding angles are equal in measure. Two triangles with identical measures are sometimes

said to be *identical.*

Note that for two triangles to have identical measures, all six corresponding measures (i.e., 3 angle measures and 3 length measures) must be the same.)

- **Two Angles and the Included Side Condition** (Two triangles satisfy the *two angles and the included side condition* if there is a triangle correspondence between the two triangles such that two pairs of corresponding angles are each equal in measure and the pair of corresponding included sides are equal in length.)
- **Two Angles and the Side Opposite a Given Angle Condition** (Two triangles satisfy the *two angles and the side opposite a given angle condition* if there is a triangle correspondence between the two triangles such that two pairs of corresponding angles are each equal in measure and one pair of corresponding sides that are both opposite corresponding angles are equal in length.)
- **Two Sides and the Included Angle Condition** (Two triangles satisfy the *two sides and the included angle condition* if there is a triangle correspondence between the two triangles such that two pairs of corresponding sides are each equal in length and the pair of corresponding included angles are equal in measure.

Familiar Terms and Symbols

- Adjacent Angles
- Angles at a Point
- Angles on a Line
- Complementary Angles
- Right Rectangular Prism
- Supplementary Angles
- Vertical Angles

Research-Based Effective Teaching Strategies	21st Century Learning Skills
Task/Activities that solidifies mathematical concepts Use questioning techniques to facilitate learning	Teamwork and Collaboration Initiative and Leadership Curiosity and Imagination Innovation and Creativity
Reinforcing Effort, Providing Recognition Practice , reinforce and connect to other ideas within mathematics	Critical thinking and Problem Solving Flexibility and Adaptability
Promotes linguistic and nonlinguistic representations	Effective Oral and Written Communication
Cooperative Learning Setting Objectives, Providing Feedback	Accessing and Analyzing Information

Varied opportunities for students to communicate mathematically Use technological and /or physical tools	

Formative Assessment	Summative Assessment	Technology
Short constructed responses Extended responses Checks for understanding Exit tickets Teacher observation Projects Timed Practice Test – Multiple Choice & Open-Ended Questions	End of Unit Assessment	NJ CORE Annenberg Learning : Insight into Algebra 1 Mathematics Assessment Projects Get the Math Achieve the Core Webmath.com sosmath.com Mathplanet.com Interactive Mathematics.com Illustrative Mathematics Inside Mathmatics.org Asia Pacific Economic Cooperation : :Lesson Study Videos Genderchip.org Interactive Geometry Mathematical Association of America National Council of Teachers of